Optimal Whereabouts Search for a Moving Target

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This paper shows that solving the optimal whereabouts search problem for a moving target is equivalent to solving a finite number of optimal detection problems for moving targets. This generalizes the result of Kadane [1971] for stationary targets.

SUPPOSE WE wish to find an object, the target, which is moving according to known probability laws. The search is assumed to last until time T, where time is discrete ($t = 0, \cdots, T$) or continuous ($0 \le t \le T$). Such a search may have a number of purposes. A common purpose is to maximize the probability of detection by time T. In this case the search is called a *detection* search. A second purpose is to localize the target to within one cell out of a finite number of cells defined by a grid system of the searcher's choice. In this case the searcher may succeed by detecting the target up to time T, or failing that, the searcher may guess one cell for the target's location at time T. If the guess is correct then the searcher also succeeds. This is called a *whereabouts* search. A third type is *surveillance* search in which one seeks to maximize the probability of correctly stating which cell contains the target at time T. The difference between a surveillance search and a whereabouts search is that a detection before time T in a surveillance search does not end the search. It merely helps to locate the target at time T.

This paper shows that the optimal whereabouts search plan, i.e., an allocation of search effort and a choice of cell to guess, may be found by solving a finite number of optimal detection problems for a moving target, one for each cell in the grid. Having shown this we discuss how to use the optimal detection search algorithms of Brown [1980] Stone et al. [1978], and the bounds given by Washburn [1981] to compute optimal whereabouts search plans. In the case of a stationary target, as studied by Mela

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[1961], Tognetti [1968], and Kadane [1971] optimal whereabouts search and optimal surveillance search coincide because, once the target is detected, its location is known with certainty from then on (and in particular at time T). In addition to the results mentioned above, optimal detection search for a moving target has been studied by Stone [1977, 1979], Stone and Richardson [1974], Persiheimo [1977], Saretsalo [1973], Hellman [1972], Dobbie [1974], Pollock [1970], Stewart [1979], and Washburn [1980]. Generalizations of the optimal detection search problem have been studied by Discenza and Stone [1981], and Stromquist and Stone [1981].

In the case where false targets are present very few results have been obtained even when the target is stationary; see Chapter VI of Stone [1975], Richardson [1973], and Barker and Belkin [1977]. In this paper, we do not consider the problem of false targets.

Whereabouts search might be used in a search and rescue situation in which it is known that those lost can survive no longer than T units of time in the environment. If the searcher finds the lost party in some search of a cell, the rescue can be effected immediately. If after time T the party has not been found, the rescue effort is assigned to some cell and will be successful if the lost party is in fact in that cell. Similarly it might be used in a military context in which the object is destroyed as soon as it is found, or after time T, a weapon is fired into the cell if the target has not been found. By contrast, if it is desired to know where the object is at time T, without taking any action if it is found before T, a surveillance search problem results.

1. CHARACTERIZATION OF OPTIMAL WHEREABOUTS SEARCH IN TERMS OF OPTIMAL DETECTION SEARCH

Kadane shows that solving the optimal whereabouts search for a stationary target may be reduced to solving J optimal detection problems for a stationary target where J is the number of cells in the grid of the target's probability distribution. In this section we generalize that result to moving targets.

Our results apply to targets moving in discrete or continuous space or time. Let $\{X_t, 0 \le t \le T\}$ be the stochastic process representing the target's motion. For discrete time, we shall understand $0 \le t \le T$ to mean $t = 0, 1, \cdots, T$ in order to state results simultaneously for continuous and discrete time. The search space may be continuous or discrete, but the searcher must specify a *whereabouts grid* of cells for time T. The size of these grid cells corresponds to the degree of localization required for the objective of the search. For example, the cells may be of the size of the lethal area of a weapon to be fired at time T. The whereabouts grid has no connection with the target motion which may be in continuous space or through a grid of cells entirely different from the whereabouts grid. Let the whereabouts grid have I cells numbered $i = 1, 2, \cdots, I$.

Let Ψ be the class of allowable detection search plans. A *whereabouts search plan* is comprised of both a search plan $\psi \in \Psi$ with which to try to detect the target and a whereabouts cell i in the whereabouts grid which is guessed to contain the target if the detection search fails. Let $S_T[\psi, i]$ be the probability of success using the whereabouts search plan (ψ, i). We seek an *optimal whereabouts* plan (ψ^*, i^*), i.e., a plan such that $\psi^* \in \Psi$, $1 \leq i^* \leq I$, and

$$S_T[\psi^*, i^*] = \max\{S_T[\psi, i] : \psi \in \Psi \text{ and } 1 \leq i \leq I\}.$$

Let $C(i)$ denote the ith cell in the whereabouts grid and let $X^i = \{X_t, 0 \leq t \leq T \mid X_T \notin C(i)\}$ denote the target process obtained by conditioning on the target not being in $C(i)$ at time T. Specifically, one obtains X^i by removing all sample paths which end in $C(i)$ at time T and then renormalizing the probability measure on the remaining paths so that it sums (or integrates) to 1. Let $P_T^i[\psi]$ be the probability of detecting the target with plan ψ given that the target is not in cell i at time T. Then

$$S_T[\psi, i] = P_T^i[\psi](1 - \Pr\{X_T \in C(i)\}) + \Pr\{X_T \in C(i)\}. \quad (1)$$

Given that we choose to guess cell i at time T, it is clear that we should choose the detection search plan ψ^i to use with i so that ψ^i maximizes P_T^i. Since I is finite it is also clear that we can find the optimal whereabouts plan by performing I optimizations to determine (ψ^{i*}, i^*), such that

$$S_T[\psi^{i*}, i^*] = \max_{1 \leq i \leq I} S_T[\psi^i, i].$$

Since finding ψ^i is simply finding an optimal detection plan for a target moving according to the stochastic process X^i, we have shown that the optimal whereabouts problem reduces to solving I moving target detection problems. We state this observation as a theorem.

THEOREM. *Let $\psi^i \in \Psi$ be an optimal detection plan for the target motion process X^i, i.e.,*

$$P_T^i[\psi^i] = \max\{P_T^i[\psi] : \psi \in \Psi\}.$$

Then an optimal whereabouts search plan is (ψ^{i*}, i^*) where

$$S_T[\psi^{i*}, i^*] = \max_{1 \leq i \leq I} S_T[\psi^i, i].$$

Observe that there are no restrictions on the class Ψ of detection plans from which we are allowed to choose: it can be a class of search paths or a collection of functions which specify the allocation of search effort in space and time. This result may also be extended to whereabouts problems in which the searcher is allowed to choose $n \geq 1$ whereabouts cells for the target's location if the detection search fails.

When time is discrete, one can consider each of the $T + 1$ time periods as a detection opportunity and the whereabouts guess at the end of the search as the $T + 2$nd opportunity. However, one must model this extra opportunity by using a detection function which may differ from the one which applies to the first $T + 1$ opportunities.

If there is an algorithm for solving all possible detection search problems, it can be used to solve the optimal whereabouts problem. In the next section, we assume an exponential detection function and infinitely divisible effort for the $T + 1$ time periods. This type of detection problem is solvable by the algorithms of Brown or Stone et al. However, when the whereabouts search is added as a $T + 2$nd time period, all effort must be placed in only one cell and is no longer infinitely divisible. As a result the probability of detection in this extended detection search is no longer concave and the conditions of Brown or Stone [1979] are no longer sufficient for optimality. In this case, it is known (see Washburn [1980]) that the obvious modification of Brown's algorithm can converge to a plan which is not optimal.

Since the whereabouts search adds a discrete effort search problem for the $T + 2$nd search stage, it involves a discrete optimization not solvable by existing optimal detection search algorithms. To account for this we present in the next section a procedure similar to branch-and-bound.

2. ALGORITHMS FOR COMPUTING OPTIMAL WHEREABOUTS PLANS

Using the result of the theorem we now show how one may modify the algorithm of Brown or Stone et al. (both of which are designed to compute optimal detection plans) to compute optimal whereabouts plans.

Assume that there is a grid of J cells in the plane over which search is allocated. This is the *search grid*. A search plan ψ is a non-negative function of space and time such that

$$\psi(j, t) = \text{effort placed in cell } j \text{ at time } t \text{ for } j = 1, \cdots, J,\ t = 0, \cdots, T.$$

Effort $m(t)$ is available at time t, which cannot be transferred from one time period to another. We are thus restricted to the class $\Psi(m)$ of plans ψ such that

$$\textstyle\sum_{j=1}^{J} \psi(j, t) = m(t) \quad \text{for } t = 0, \cdots, T.$$

The detection function is exponential with sweep width which may vary over space. Let $W(j)$ and $A(j)$ be the sweep width and area of the jth cell for $j = 1, \cdots, J$. We assume

$$P_T[\psi] = E[1 - \exp(-\textstyle\sum_{s=0}^{T} W(X_s)\psi(X_s, s)/A(X_s))] \qquad (2)$$

where E indicates expectation over the sample paths of the process and X_s is the cell that the target is in at time s.

In order to modify these algorithms, it is most efficient to consider minimizing the whereabouts failure probability

$$F_T[\psi, i] \equiv 1 - S_T[\psi, i] = (1 - P_T^i[\psi])(1 - \Pr\{X_T \in C(i)\}).$$

From the above equation, we can see that $F[\psi, i]$ is the joint probability of failing to detect the target with plan ψ and of the target being outside the cell $C(i)$ at time T.

We first discuss how to modify Brown's algorithm. When target motion is modeled by a mixture of discrete time and space Markov processes and the detection function is exponential, Brown's algorithm provides an extremely efficient method of computing optimal detection plans.

Let $c(j)$ for $j = 1, \cdots, J$ indicate the cells in the search grid. The target moves among the cells in the search grid and each cell $C(i)$, $i = 1, \cdots, I$ in the whereabouts grid is composed of an integral number of cells from the search grid. To adapt Brown's algorithm, we modify his survive matrix by changing the initialization so that (using Brown's notation)

$$\text{survive } (c(j), T, \psi) = \begin{cases} 0 & \text{if } c(j) \in C(i) \\ 1 & \text{if } c(j) \notin C(i) \end{cases} \quad \text{for } j = 1, \cdots, J.$$

This sets the probability equal to 0 on all paths which end in $C(i)$ at time T. Applying the algorithm with this modified survive matrix yields a plan ψ^* which minimizes the probability of failing to detect the target and of the target being outside the set $C(i)$ at time T. That is, Brown's algorithm will minimize $F[\psi, i]$ over all $\psi \in \Psi$, so that $\psi^* = \psi^i$. By performing the above modification for each cell in the whereabouts grid and choosing the cell and associated plan with the highest success probability, $S_T[\psi^i, i] = 1 - F_T[\psi^i, i]$, the whereabouts problem is solved for mixtures of Markov chains.

Brown's algorithm is iterative and converges to the optimal plan in an infinite number of steps. However, after each iteration in the algorithm one can obtain a lower bound, $\underline{S}_T[\psi^i, i]$, i.e., the probability of success for the plan obtained from that iteration, and an upper bound, $\bar{S}_T[\psi^i, i]$, by the method of Washburn [1981]. With these bounds, one can use a branch-and-bound technique as follows.

Specify a tolerance $\epsilon > 0$ for how close to the success probability of the optimal plan one wishes to come. Choose an initial whereabouts cell i' and iterate until $\bar{S}_T[\psi^{i'}, i'] - \underline{S}_T[\psi^{i'}, i'] < \epsilon$. For each succeeding choice of a whereabouts cell $C(i)$, check after each iteration of the algorithm to see if either $\bar{S}_T[\psi^i, i] \le \underline{S}_T[\psi^{i'}, i']$ *or* $\underline{S}_T[\psi^i, i] \ge \bar{S}_T[\psi^{i'}, i']$. If the former occurs, retain $(\psi^{i'}, i')$ as the candidate optimal plan and stop the iteration. If the latter occurs, then replace i' with i and continue iterating until the tolerance ϵ is met for the new candidate i'. If the tolerance ϵ has been reached, and one still cannot decide between the two possible solutions on the basis given above (i.e., the tolerance interval for one plan overlaps the other), then choose between them in an arbitrary fashion. This guarantees a solution within the desired tolerance.

If the target motion is more general than a mixture of discrete space and time Markov processes, then one can modify the algorithm in Chapter IV of Stone et al. [1978] as follows.

Sort the sample paths of the target motion process by the whereabouts cell in which they are located at time T. Choose a whereabouts cell $C(i)$. Remove the paths ending in $C(i)$ from the file of sample paths and solve for ψ^*, the optimal detection search plan, using the remaining paths. As above, this plan will minimize $F_T[\cdot, i]$ so that $\psi^* = \psi^i$, and one may use the above branch-and-bound technique to find the optimal whereabouts plan (ψ^{i^*}, i^*) to the desired tolerance.

3. EXAMPLES OF OPTIMAL WHEREABOUTS PLAN

We give two examples of optimal whereabouts search plans. The first example shows that even when the detection function is exponential and the sweep width the same in all cells, it is not necessarily optimal to choose the cell with the highest prior probability at time T for the whereabouts cell. The second example shows a situation in which the optimal whereabouts cell is the one with the highest probability at time T when no search is applied at any time period.

For the examples presented in this section, the whereabouts grid coincides with the search grid. The probability of detection using plan ψ is given by Equation (2).

Example 1. For stationary whereabouts searches, Kadane shows that when the cost and detection function are the same over all the cells in the whereabouts grid, then it is optimal to choose the cell with the highest prior probability for the whereabouts cell. One might conjecture that this result would generalize to whereabouts searches involving moving targets, by choosing as the whereabouts cell the one with the highest probability at time T when no search is applied. We now show this conjecture to be false.

Let the search and whereabouts grid consist of two cells. Time is discrete and there are two time periods for search, $t = 0, 1$. There are three possible target paths. The paths and their associated probabilities are shown in Table I. The notation $\omega_1 = (1, 1)$ means that path 1 is in cell 1 at time 0 and remains in cell 1 at time 1. Table I also shows the target distributions if no search is conducted, note that cell 2 has the highest probability at time 1. There are two units of search effort available at each time period, and $W(j) = A(j) = 1$ for $j = 1, 2$.

Let F^j be the failure probability for the whereabouts plan which chooses cell j for the whereabouts cell and allocates its detection search optimally for the process X^j which has the sample paths ending in cell j removed. Then $F^j = 1 - S_T[\psi^j, j]$.

Suppose we choose 1 for the whereabouts cell. Since ω_3 is the only path not ending in cell 1, the plan ψ^1 puts 2 units of effort in cell 2 at time 0 and time 1. Thus, $F^1 = (\frac{1}{2} + \epsilon)e^{-4}$.

If we choose 2 for the whereabouts cell, then ψ^2 places all the effort for time $t = 1$ in cell 1. By Brown or Theorem 2 of Stone [1979], we find the optimal allocation at time 0 by computing the posterior distribution at time 0 (using only ω_1 and ω_2), given failure to detect at time 1, and allocating the effort for time 0 to be optimal for this posterior distribution. If ϵ is small enough, then ψ^2 will place all effort in cell 2 at time 0 and

TABLE I

a. Target Paths and Probabilities[a]

Path	Probability
$\omega_1 = (1, 1)$	$\epsilon < \frac{1}{4}$
$\omega_2 = (2, 1)$	$\frac{1}{2} - 2\epsilon$
$\omega_3 = (2, 2)$	$\frac{1}{2} + \epsilon$

b. Target Distributions

Target Distribution at Time 0		Target Distribution at Time 1	
Cell	Probability	Cell	Probability
1	ϵ	1	$\frac{1}{2} - \epsilon$
2	$1 - \epsilon$	2	$\frac{1}{2} + \epsilon$

[a] *Note*: $\epsilon < \frac{1}{4}$.

$$F^2 = (\tfrac{1}{2} - 2\epsilon)e^{-4} + \epsilon e^{-2}.$$

Since $F^2 - F^1 = \epsilon e^{-2}(1 - 3e^{-2}) > 0$, it follows that choosing cell 1 yields the lower failure probability (higher success probability) and that the optimal whereabouts plan chooses cell 1 for the whereabouts cell rather than the higher probability cell 2.

Observe that no matter how small ϵ is, cell 2 has the larger probability of containing the target at time 1 given no search. By making ϵ small enough the searcher is forced to place all effort at time 0 in cell 2 regardless of the whereabouts cell chosen. However, searching in cell 2 at time 0 causes cell 1 to be the high probability cell at time 1 given failure to detect at time 0. Thus cell 1 rather than 2 becomes the optimal whereabouts cell.

Example 2: *Uniform Sweep Width.* This example shows a situation in which the cell having the highest probability of containing the target at time T, if no search takes place, is the optimal one to choose for the whereabouts cell.

The target distribution at time 0 is bivariate normal with center at 30°10′N 30°10′W. The major axis is oriented east-west, and the standard deviation along the major and minor axes is 50 and 30 nautical miles, respectively. At time $t = 3$, the target's distribution is circular normal with center at 28°10′N, 30°10′W and standard deviation 10 nautical miles along any axis. The target paths are obtained by making an independent draw for the target's position at time $t = 0$ and $t = 3$ from the above

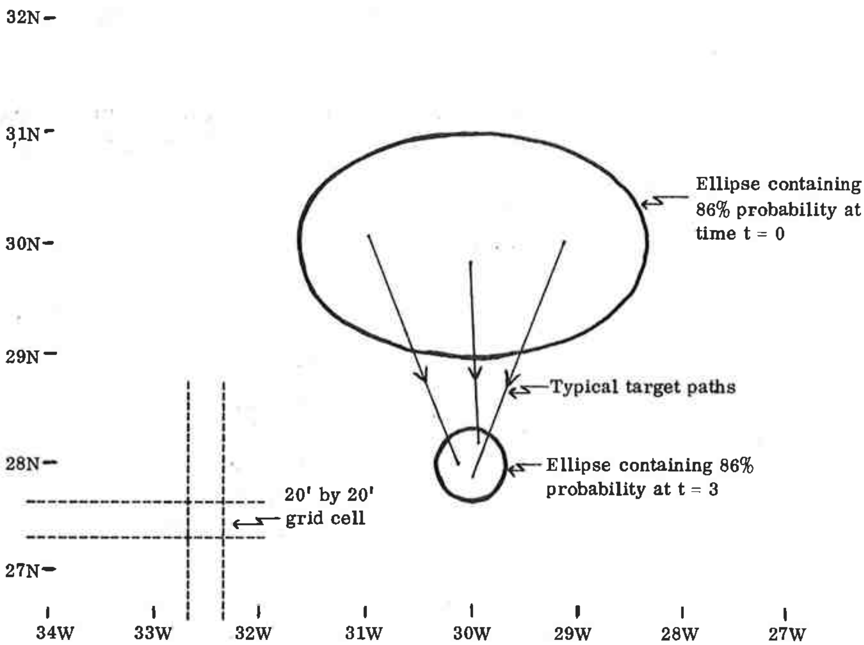


Figure 1. Target motion assumptions for Example 2.

distributions. The target then follows a constant course and speed between these points. Four thousand sample paths were drawn to represent the target motion process. This motion model applies to the situation in which one knows that the target will pass through a straight or choke point at a certain time.

Figure 1 illustrates the target motion assumptions and shows some typical target paths. Both the search and whereabouts grid consists of cells which are 20′ by 20′ as indicated in the figure. Thus, the area $A(j)$ of cell j is approximately 400 (naut m)^2 and will vary slightly over the grid as latitude changes. The sweep width $W(j) = 1$ nautical mile for all cells j, and there are 1,500 units of search effort, i.e., nautical miles of

	0	0	1	0	0
	0	27	96	31	0
28°N —	2	133	426	126	3
	0	30	91	31	0
	0	0	0	0	0
				30°W	

Figure 2. Target distribution in whereabouts grid at time 3 when no search is applied during any time period. *Note:* Entries are probabilities multiplied by 10^3.

search track length, available at each time period. In subsequent figures, we will show only the region of the grid in which the target location probabilities are positive.

Figure 2 shows the probability distribution in the whereabouts grid at time 3 with no search having taken place at any time period. The numbers in the cells are the probabilities that the target is in that cell multiplied by 10^3. (Due to round off error, these numbers do not sum to exactly 1,000). The algorithm used to find the optimal whereabouts plan is a modification of one given in Chapter IV of Stone et al. The algorithm proceeded by choosing the highest probability cell in this grid and computing the optimal plan given this choice of whereabouts cell to within a tolerance of 0.001. This tolerance was reached at 3 iterations. An iteration consists of one complete cycle through the time periods. For convenience in referring to the cells in the whereabouts grid in Figure 2,

TABLE II

CONVERGENCE OF ALGORITHM TO OPTIMAL WHEREABOUTS PLAN[a]

Whereabouts Cell Chosen	Success Probability		No. of Iterations
	Lower bound	Upper bound	
13	0.8619	0.8623	3
12	0.8370	0.8447	2
14	0.8358	0.8432	2
18	0.8303	0.8382	2
8	0.8292	0.8340	2
9	0.7996	0.8470	1
19	0.8010	0.8470	1
7	0.7979	0.8447	1
17	0.7980	0.8436	1
15	0.7843	0.8303	1
11	0.7849	0.8308	1
23	0.7837	0.8300	1

[a] *Note*: Solution tolerance = 0.001.

	0	0	0	0	0	0	0	3	0	3	0	0	0	0	0	0	0	0	0
	0	0	0	3	5	3	5	5	3	8	18	15	0	0	0	3	3	0	0
	0	0	5	5	10	18	35	35	45	53	58	45	33	13	18	3	3	0	3
31°N —	0	0	5	18	30	58	75	145	155	148	178	90	95	55	38	23	0	5	8
	3	10	8	53	68	123	187	205	277	270	245	232	180	108	65	28	23	8	0
	3	13	8	35	78	118	227	290	332	370	362	302	216	133	63	38	23	8	5
30°N —	0	8	8	48	65	120	192	245	257	275	290	232	173	90	98	30	25	5	8
	0	5	15	23	35	55	135	105	123	145	135	108	78	68	48	18	10	3	0
	0	0	3	5	18	20	40	35	45	48	40	33	30	18	10	3	3	0	0
29°N —	0	0	0	0	0	3	0	10	18	10	23	15	3	5	3	0	3	0	0
	0	0	0	0	0	0	0	0	0	5	3	0	0	0	0	0	0	0	0

33°W 32°W 31°W 30°W 29°W 28°W

OPTIMAL ALLOCATION OF EFFORT FOR TIME 0

31°N —	0	0	0	0	0	18	0	0	0	0
	0	144	119	78	75	0	24	114	31	0
30°N —	21	66	68	79	0	62	66	21	147	0
	54	23	96	20	45	12	65	0	0	48

31°W 30°W 29°W

PROBABILITY OF DETECTION BY THE END OF TIME 0 = 0.09

Figure 3A. Uniform sweep width example. Target distribution before search at time 0. *Note:* Entries are probabilities multiplied by 10^4.

we number them starting with the bottom row and numbering from left to right. The highest probability cell is cell 13. Table II shows the convergence of the algorithm, which chooses cells in the order of highest to lowest probability in Figure 2. Since the high probability cell in Figure

	0	0	3	5	8	8	11	25	5	0	3	0	0
30°N —	0	5	16	49	82	126	144	127	77	41	16	3	5
	5	16	90	153	243	425	467	465	287	174	44	25	5
	11	16	98	244	425	602	723	680	429	225	79	52	8
29°N —	3	25	68	174	356	389	474	488	239	192	83	25	5
	0	3	5	57	85	109	164	112	71	49	8	3	0
	0	0	0	3	5	11	14	25	5	0	3	0	0
	0	0	0	0	0	0	3	0	0	0	0	0	0

32°W 31°W 30°W 29°W

OPTIMAL ALLOCATION OF EFFORT FOR TIME 1

30°N —	0	32	127	147	182	39	0
	18	6	163	119	188	8	5
29°N —	0	169	97	59	140	0	2

31°W 30°W 29°W

PROBABILITY OF DETECTION BY THE END OF TIME 1 = 0.25

Figure 3B. Uniform sweep width example. Target distribution before search at time 1. *Note:* Entries are probabilities multiplied by 10^4.

	Ø	1	1	3	2	Ø	Ø
29°N —	1	17	50	72	50	13	2
	5	46	127	183	141	50	7
	2	20	49	82	53	17	1
	Ø	Ø	2	4	1	Ø	Ø
	31°W			30°W			

OPTIMAL ALLOCATION OF EFFORT FOR TIME 2

29°N —	Ø	125	62	Ø
	282	388	342	13
	Ø	232	55	Ø
		30°W		

PROBABILITY OF DETECTION BY THE END OF TIME 2 = 0.52

Figure 3C. Uniform sweep width example. Target distribution before search at time 2. *Note:* Entries are probabilities multiplied by 10^3.

2 is the optimal whereabouts cell for this problem, the initial guess is optimal and the remaining guesses are eliminated very quickly after one or two iterations (see Table II). The optimization algorithm required 702 CPU seconds on a Prime 400 minicomputer. This time does not include the time necessary to generate the sample paths for the target motion process.

Figure 3*A–D* shows the target distribution before search and the optimal allocation of search at times $t = 0, 1, 2, 3$ for this example. The rectangle in the target distribution outlines the region in which search is placed at that time period. Observe that no effort is placed in the high

	Ø	Ø	1	Ø	Ø
	Ø	29	100	34	Ø
28°N —	3	131	419	124	2
	1	30	94	31	1
	Ø	Ø	Ø	Ø	Ø
			30°W		

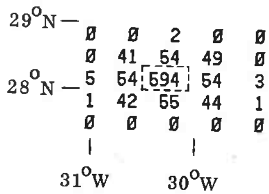
OPTIMAL ALLOCATION OF EFFORT FOR TIME 3

	Ø	337	Ø
28°N —	433	Ø	415
	Ø	315	Ø
		30°W	

PROBABILITY OF DETECTION BY THE END OF TIME 3 = 0.66

Figure 3D. Uniform sweep width example. Target distribution before search at time 3. *Note:* Entries are probabilities multiplied by 10^3.

probability cell at time 0 and time 3 whereas substantial amounts of effort are placed in the high probability cell at times 1 and 2. Since the high probability cell at $t = 3$ is also the whereabouts cell, it is clear that no search should be placed in this cell at time 3. The fact that no effort is placed in the high probability cell at time 0 is surprising although consistent with Example One of Brown which shows a low but nonzero amount of effort in the high probability cell at time 0 for an optimal detection search involving a constrained or focused motion. Figure 4 shows the target distribution given failure of the detection search.



PROBABILITY OF SUCCESS IN WHEREABOUTS SEARCH = 0.86

Figure 4. Target distribution following unsuccessful search at time 3 in uniform sweep width example. *Notes:* (1) Entries are probabilities multiplied by 10^3. (2) The cell enclosed by the dashed lines is the optimal whereabouts cell. If the target is not detected by the end of time 3, this cell is guessed to contain the target.

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REFERENCES

Barker, W. H., and B. Belkin. 1977. *Information Processing to Maintain Localization in ASW Surveillance*, Daniel H. Wagner, Associates Report to the Office of Naval Research/Naval Analysis Programs (DDC No. AD A048190).

Brown, S. S. 1980. Optimal Search for a Moving Target in Discrete Time and Space. *Opns. Res.* **28,** 1275–1289.

DISCENZA, J. H., AND L. D. STONE. 1981. Optimal Survivor Search with Multiple States. *Opns. Res.* **29,** 309–323.

DOBBIE, J. M. 1974. A Two-Celled Model of Search for a Moving Target. *Opns. Res.* **22,** 79–92.

HELLMAN, O. 1972. On the Optimal Search for a Randomly Moving Target. *SIAM J. Appl. Math.* **22,** 545–552.

KADANE, J. B. 1971. Optimal Whereabouts Search. *Opns. Res.* **19,** 894–904.

MELA, D. F. 1961. Information Theory and Search Theory as Special Cases of Decision Theory. *Opns. Res.* **9,** 907–909.

PERSIHEIMO, U. 1977. On the Optimal Search for a Target Whose Motion Is Conditionally Deterministic with Stochastic Initial Conditions on Location and Parameters. *SIAM J. Appl. Math.* **32,** 105–114.

POLLOCK, S. M. 1970. A Simple Model of Search for a Moving Target. *Opns. Res.* **18,** 883–903.

RICHARDSON, H. R. 1973. *ASW Information Processing and Optimal Surveillance in a False Target Environment,* Daniel H. Wagner, Associates Report to the Office of Naval Research (DDC No. AD A002254).

SARETSALO, L. 1973. On the Optimal Search for a Target Whose Motion Is a Markov Process. *J. Appl. Prob.* **10,** 847–856.

STEWART, T. J. 1979. Search for a Moving Target when Searcher Motion Is Restricted. *Comput. Opns. Res.* **6,** 129–140.

STONE, L. D. 1975. *Theory of Optimal Search.* Academic Press, New York.

STONE, L. D. 1977. Search for Targets with Generalized Conditionally Deterministic Motion. *SIAM J. Appl. Math.* **33,** 456–468.

STONE, L. D. 1979. Necessary and Sufficient Conditions for Optimal Search Plans for Moving Targets. *Math. Opns. Res.* **4,** 431–440.

STONE, L. D., AND H. R. RICHARDSON. 1974. Search for Targets with Conditionally Deterministic Motion. *SIAM J. Appl. Math.* **27,** 239–255.

STONE, L. D., S. S. BROWN, R. P. BUEMI AND C. R. HOPKINS. 1978. *Numerical Optimization of Search for a Moving Target.* Daniel H. Wagner, Associates Report to Office of Naval Research.

STROMQUIST, W. R., AND L. D. STONE. 1981 Constrained Optimization of Functionals with Search Theory Applications. *Math. Opns. Res.* (to appear).

TOGNETTI, K. P. 1968. An Optimal Strategy for a Whereabouts Search. *Opns. Res.* **16,** 209–211.

WASHBURN, A. R. 1980. On a Search for a Moving Target. *Naval Res. Logist. Quart.* **27,** 315–322.

WASHBURN, A. R. 1981. An Upper Bound Useful in Optimizing Search for a Moving Target. *Opns. Res.* **29,** 1227–1230.