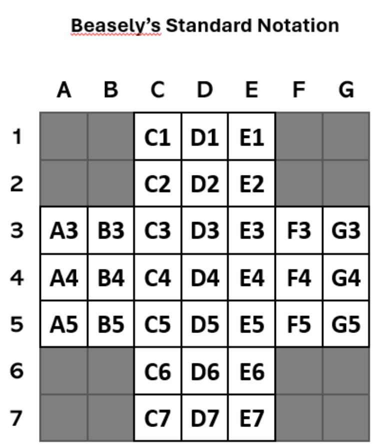
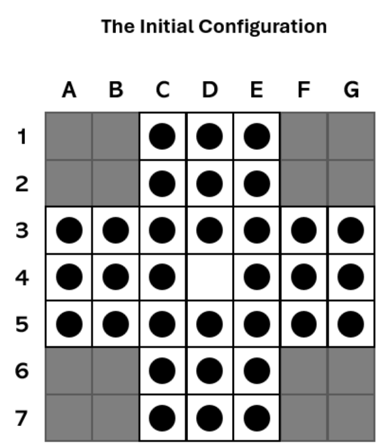
JMM 2026 – Metron's Daily Math Problems – Sheet A (Sunday, January 4)

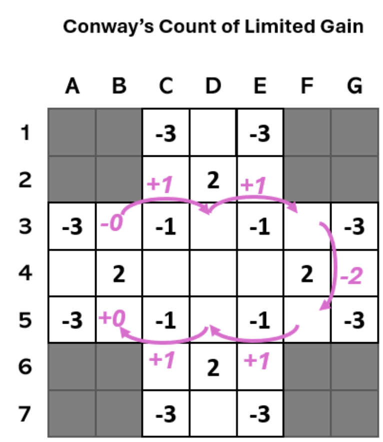
A1) "Each snap makes one more piece" is a six-word proof. It takes 24 snaps to break 1 piece into 25. This structure is so simple that it is difficult to see. Instead, the question lures you into considering superfluous geometric details.

A2) The minimum number is 8. Lighting the main diagonal of the grid suffices: all other diagonals light up in succession from this first one. If we picture each bulb as a (lit or unlit) square of the 8 × 8 grid, then minimality is due to the perimeter of the lit region never increasing. When a bulb with exactly L lit neighbors turns on, the lit perimeter:
(a) decreases by L due to one edge on each of those L bulbs no longer being on the perimeter, and
(b) increases by at most $4 - L$ due to some edges on the newly lit bulb (i.e., those not adjacent to the L lit neighbors) potentially being on the perimeter.
Therefore, the total increase of the lit perimeter is at most $4 - 2L$. This is nonpositive because a bulb can only turn on if $L \geq 2$. Any initial configuration with 7 bulbs or fewer has a perimeter of at most 28, so it cannot evolve into the final configuration, which has perimeter 32.

A3) Peg solitaire was the Rubik's Cube of the early 1700s. Knowing how to solve the locally fashionable version marked you as one of the cognoscenti. As with the Rubik's Cube, many people learned solutions from printed pamphlets. Here is a modern pamphlet for the 33-hole English cross version which presents Bergholt's optimal 18-move solution.







Any move's net effect is to increase the board value by no more than +2

In his book *The Ins and Outs of Peg Solitaire,* mathematician and puzzle enthusiast John Beasley provides a comprehensive survey of the mathematical treatments of Peg Solitaire, including a proof that Bergholt's 18-move solution is optimal. This is done by showing that a 17-move solution cannot exist (and therefore smaller solutions are likewise impossible).

We break the hypothetical 17-move solution into 5 parts: the first move, the last move, the penultimate move, 8 outside corner clearing moves, and 6 free moves. Using Conway's count of limited gain (which assigns values to squares in the puzzle: see the figure above), the initial board has a value of –20 points, and our target, the inverted board, has a value of 0 points. The first and last moves each lose 2 points, since both must be a single jump to the center. The penultimate move must set up this jump, and gains at most 1 point (e.g. C2, C4: +1). The corner clearing is similarly bounded such that each of the 8 moves gains no more than 2 points. This leaves at least 7 points for the 6 free moves to accumulate. The remainder of the proof consists of working through the possible uses of these free moves. Beasley demonstrates that the board cannot be cleared without running out of free moves before meeting the point requirement. Thus, the 17-move solution is impossible. Bergholt's 18-move solution is indeed optimal.

To answer the final question, if we count individual jumps as moves, then, by the same reasoning as (A1), all solutions are equally good. "Each jump eats one more peg," so it takes 31 jumps to reduce 32 pegs to 1.